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Sun-Perturbed Earth-to-Moon Transfers with Ballistic Capture

Edward A. Belbruno* and James K. Miller†
Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California 91109

A method is described for constructing a new type of low energy transfer trajectory from the Earth to the moon leading to ballistic capture. This is accomplished by utilizing the nonlinear Earth-moon-sun perturbations on a point mass in three dimensions. The interaction of the gravitational fields of the bodies defines transition regions in the position-velocity space where the dynamic effects on the point mass tend to balance. These are termed weak stability boundaries. The transfer is obtained by the use of trajectories connecting the weak stability boundaries. It uses approximately 18% less ΔV than the Hohmann transfer to insert a spacecraft into a circular orbit about the moon. The use of this transfer has recently been demonstrated by Japan's Hiten spacecraft, which arrived at the moon on October 2, 1991. Application of the transfer method is also made to the Lunar Observer Mission.

Nomenclature

DV = change of velocity at $\hat{x}$
e = eccentricity
G = gravitational constant
H_k = Kepler energy with respect to E (Earth), M (moon) for $k = 3, 4$, respectively
h = altitude above the moon
l = radial line extending from central mass
m = point mass or spacecraft
m_k = mass of m, S (sun), E, M for $k = 1, 2, 3, 4$, respectively
P = half-plane along line l
$\mathbb{R}^3$ = three-dimensional real vector space
r^* = stability breakdown distance relative to E
r_k^* = stability breakdown distance relative to E, M for $k = 3, 4$, respectively
T = time spacecraft is at $\hat{x}$
t_F = time of capture
u = approach direction to the moon
V_C = velocity at lunar capture
V_c = circular velocity about the Earth at $h = 167$ km
V_I = velocity at Earth injection
V_∞ = hyperbolic excess velocity
WSB$_k$ = weak stability boundary of E, M for $k = 3, 4$, respectively
x_{I} = three-dimensional position part of Φ_{I}
x_{II} = three-dimensional position part of Φ_{II}
x_k = position of m, S, E, M for $k = 1, 2, 3, 4$, respectively, with components (x_{k1}, x_{k2}, x_{k3})
$\hat{x}$ = three-dimensional position part of $\hat{\Phi}$
α = angle of velocity direction relative to the central mass on plane perpendicular to l
β = angle between EM and ES lines
Γ = trajectory from E to M by linking Φ_{I} and Φ_{II}
ΔV = change in velocity
θ, φ = spherical angles about central mass
μ = Gm_4
Φ_C = capture state of m relative to M at $t = t_F$
Φ_E = injection state of m relative to E at $t = t_0$
Φ_{I} = trajectory for m from Φ_E to $\hat{\Phi}$
Φ_{II} = trajectory for m from $\hat{\Phi}$ to Φ_C
$\Phi_k(t)$ = trajectory of m in phase space relative to E, M for $k = 3, 4$, respectively
$\Phi(t)$ = solution curve, or trajectory, in phase space describing motion of m with six components $[x_1(t), \dot{x}_1(t)]$
$\hat{\Phi}$ = $\Phi_3(T)$

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*Consulting Staff, Mission Design Section, MS 301/140H, 4800 Oak Grove Drive; also Associated Researcher of Space Science, The Geometry Center, University of Minnesota, 1300 South Second Street, Minneapolis, MN 55454. Senior Member AIAA.

†Technical Staff, Navigation Section, 4800 Oak Grove Drive. Associate Fellow AIAA.

I. Introduction

THE classical approach to transfer to the moon from the Earth is by the well-known Hohmann transfer. This type of transfer is two body in nature. That is, it is constructed by determining a two-body Keplerian ellipse from the Earth to the moon where the two bodies are the Earth and a point mass m, e.g., a spacecraft. Such transfers have a hyperbolic excess velocity V_∞ relative to the moon which determines the ΔV required to be captured into an elliptic orbit about the moon. A method is described in this paper whereby transfers to the moon can be obtained in three dimensions that are ballistically captured at the moon, i.e., no ΔV is required to achieve an elliptic state at lunar periapsis. However, this capture is unstable and can be stabilized for a negligible amount of energy. This type of capture thus eliminates the hyperbolic excess velocity at lunar periapsis. This results in a substantial propellant savings for spacecraft. However, the time of flight from the Earth to the moon is larger than the Hohmann time of flight.

The transfer discussed in this paper is obtained by utilizing the perturbative effects of the Earth-moon-sun on m at all times to achieve the ballistic capture. The technique for doing this first requires the estimation of regions in the phase space (i.e., position-velocity space) where the perturbative effects of the Earth-moon-sun acting on m tend to balance. These regions are called weak stability boundaries (WSB) and are defined in Refs. 1 and 2. They are defined in this paper in Sec. II and are estimated by studying the breakdown of "stable motion" about a body in question. The term stable motion is also defined in Sec. II. Estimating the breakdown of stable motion along a span of different directions about a body in question in the six-dimensional phase space yields a five-dimensional region, which is the weak stability boundary where a particle m feels the gravitational attraction of the central body and the other perturbations in a nearly equal fashion. The resulting dynamics near this region are very nonlinear, and the particle m can be abruptly pulled away from the central body by the other perturbations. Such a boundary in the phase space exists about the moon, due to the effects of the Earth and sun, and about the Earth, due to the moon and sun. When projected into the three-dimensional physical space, these boundaries continuously extend from the central body out to a maximal distance along any direction from the central body. To be in the boundary at a given distance from the central body, the particle m must have the necessary velocity.

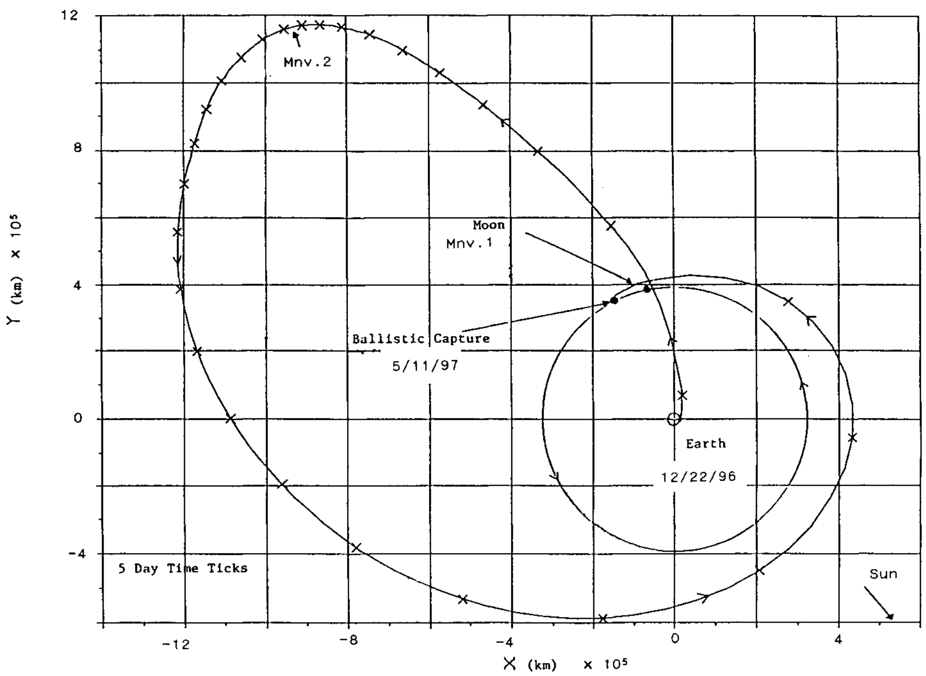

Fig. 1 Numerical simulation of a ballistic capture transfer trajectory for the Lunar Observer Mission; ecliptic plane projection, sun's direction indicated at Earth injection.

Thus, in physical space, the weak stability boundary describes a continuous three-dimensional region about the central mass with a velocity associated to each point of the region depending on the direction of motion. By its definition, the weak stability boundary is a location where escape from a central body can occur. Similarly, it is a region where capture can occur. In general, the capture is temporary. The WSB may be viewed as a more general and precise estimation of the notion of "sphere of influence." Although many definitions can be made for the sphere of influence or, more generally a transition region, the WSB is defined by observing the actual behavior of the dynamics of m about the central body due to the perturbative effects of the other bodies. This makes it possible to map out a true region where transition behavior can occur.

The method of transferring from the Earth to the moon to achieve ballistic lunar capture is described in detail in Sec. III. The idea is to leave from a given point near the Earth and fly by the moon to gain enough energy to go out to the WSB of the Earth, due to the sun and moon, at approximately four Earth-moon distances from the Earth or 1.5×10^6 km. This WSB is labeled WSB(E) for reference. When the particle m is at the WSB(E), small changes make large deviations in the motion due to the sensitivity of m while in this region. A small amount of energy can be used, i.e., via the thrusters of a spacecraft to match the initial conditions of a ballistic capture trajectory connecting the WSB(E) and the WSB of the moon, due to the sun and Earth, labeled WSB(M). The point of the WSB(M) that the trajectory goes to in its osculating periapsis has the required capture conditions so that at the given distance from the moon the trajectory has an osculating elliptic state with respect to the moon. The elliptic state is in general unstable so that the capture is temporary and should be stabilized if a more stable capture is required. Thus, the transfer can be viewed as being in two parts. The first part consists of the transfer from the Earth via a lunar flyby of the WSB(E), and the second part consists of going from the WSB(E) via a ballistic lunar capture trajectory to the WSB(M). An example of one of these transfers projected on the Earth-sun plane is shown in Fig. 1. It is described in more detail in Sec. III. The existence of a transfer of this type was first described in Refs. 2 and 3. The general idea of going beyond the Earth-moon system and then returning to the moon can be viewed as similar in principal to a classical two-body biparabolic transfer from the Earth to infinity, and then from infinity back to the moon. These are described in Ref. 4. At infinity, a zero ΔV maneuver can raise periapsis to the moon's distance from the Earth, which decreases the hyperbolic excess velocity at the moon. However, this still yields a hyperbolic excess velocity at lunar periapsis that is less than that for a Hohmann transfer.

The transfer method presented here offers two important advantages over classical approaches. The first is that going to the WSB(E) allows for a raise of periapsis for nearly zero ΔV because of the sensitivity of this region. In this sense "infinity" has been brought to a finite distance due to four-body effects. The other important advantage is that use of the WSB(M) eliminates the hyperbolic excess velocity and yields an elliptic state at lunar periapsis. The finite distance of the WSB(E) yields realistic times of flight, and the elliptic state at lunar periapsis together with the near zero maneuver at the WSB(E) yield substantial savings in ΔV for inserting a payload about the moon in a circular orbit. It is shown subsequently that this new transfer improves this ΔV over a biparabolic transfer by 14% and a Hohmann transfer 18%. The improvement over a bielliptic transfer is 37%. This is mainly due to the relatively large midcourse maneuver that a bielliptic transfer requires at the WSB(E) location. This is described in Sec. IV.

The transfer obtained for the four-body problem can be used as an initial guess in a more realistic model of the solar system where the motions of the Earth, moon, and sun are modeled with an ephemeris. Retargeting the transfer using a differential correction algorithm readily finds the transfer in this more realistic model. This is described in Sec. III. The ability to find these transfers in the more accurate modeling shows their applicability to spacecraft. This transfer has been demonstrated by the Japanese spacecraft Hiten which started in April 1991 and arrived at the moon on October 2, 1991. Hiten did not have enough propellant to reach the moon and become captured by classical methods. This is described in Sec. III. Application of this transfer for the Lunar Observer Mission is carried out in Sec. IV. Hiten was launched in January 1990 by the Institute of Space and Astronautical Science (ISAS) in Japan.

Most of the results are obtained by numerical integration due to the nonlinearities involved. Geometric results from dynamical system theory[5] are used to motivate the existence of various behaviors observed.

II. Model, Weak Stability Boundaries, and Capture Orbits

The motion of the point mass m is described by the three-dimensional four-body problem between m, sun, Earth, and moon. The only force acting between the masses is given by the Newtonian gravitational inverse square force law. The equations of motion are given by

$$m_k\ddot{x}_k = \sum_{\substack{i=1\\i\neq k}}^{4} Gm_im_k|x_i - x_k|^{-3}(x_i - x_k) \tag{1}$$

where $k = 1, 2, 3, 4$ and $x_k = (x_{k1}, x_{k2}, x_{k3}) \in \mathbb{R}^3$ is the position in inertial coordinates (x,y,z) of the point mass m_k. Therefore, this is a system of 12 second-order differential equations. The units of kilometers, kilograms, and seconds are assumed throughout unless otherwise mentioned. As will be discussed later, another model of the four-body problem will be used for applications that more accurately models the solar system.

The initial values $x_k(0)$, $\dot{x}_k(0)$ for $k = 2, 3, 4$ for the sun, Earth, and moon, respectively, are given from an epoch of interest corresponding to $t = 0$. It is assumed that a set of initial values are chosen to have a realistic starting point for the planets. The goal of this paper is to determine the initial values $x_1(0)$, $\dot{x}_1(0)$ for m to achieve a ballistic capture at the moon from a given position about the Earth.

To facilitate the construction of this transfer, the WSB is defined about the Earth due to the sun and moon, and about the moon due to the Earth and sun. As described in the Introduction, it is a region where stable motion breaks down about a central body, e.g., moon or Earth, due to other perturbations. For notation, E, M, and S represent the Earth, moon, and sun, respectively.

In what follows, motion will be defined with respect to E or M. The coordinates of the solutions of Eq. (1) with respect to E, labeled $\bar{x}_k$, are defined by the transformation $\bar{x}_k = x_k - x_3$, where E is the origin $(\bar{x}_3 = 0)$. This is a noninertial coordinate system $(\bar{x},\bar{y},\bar{z})$ where the differential equations for $\bar{x}_k$, $k = 1, 2, 4$, are

$$m_k\ddot{\bar{x}}_k = \sum_{\substack{i=1\\i\neq k}}^{4} Gm_im_k|\bar{x}_i - \bar{x}_k|^{-3}(\bar{x}_i - \bar{x}_k) - \sum_{\substack{i=1\\i\neq 3}}^{4} Gm_im_3|\bar{x}_i|^{-3}\bar{x}_i \tag{1$'$}$$

with $\bar{x}_3 = 0$. For simplicity of notation, the following convention is assumed: When referring to motion relative to E, the bars over the coordinates are dropped and the resulting coordinates x_k are not to be confused with those for system (1). The same convention is adopted for motion about M, where the transformation $\bar{x}_k = x_k - x_4$, $i = 1, 2, 3$, is used to arrive at a system similar to Eq. (1′).

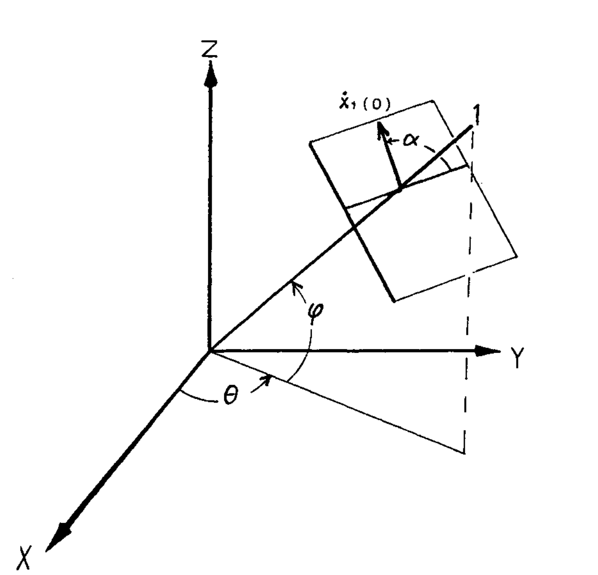

Fig. 2 Angles α, θ, φ specifying the direction of propagation of a trajectory about the Earth at the initial time $t = 0$: at $t = 0$, the x axis lies along the Earth-sun line, and β is the angle between the x axis and the Earth-moon line.

Table 1 Distance r_4^* for $e = 0$ (1 unit r_4^* = EM distance)

θ	φ	α	r_4^*
0	0	0	0.09
		$\pi/2$	0.12
		π	0.19
		$3\pi/2$	0.12
$\pi/2$	0	0	0.08
		$\pi/2$	0.23
		π	2.95
		$3\pi/2$	0.23
π	0	0	0.10
		$\pi/2$	0.11
		π	0.23
		$3\pi/2$	0.11
$3\pi/2$	0	0	0.08
		$\pi/2$	0.12
		π	0.39
		$3\pi/2$	0.12
—	$\pi/2$	0	0.09
		$\pi/2$	0.17
		π	0.09
		$3\pi/2$	0.17

Table 2 Estimation of $e(r_4^*)$

e	r_4^*
0	0.09
0.1	0.07
0.5	0.04
0.99	0.001

Stable motion about the Earth is defined as follows: Choose a radial direction D from the Earth given by the spherical angles $\theta \in [0,2\pi]$, $\varphi \in [-\pi/2,\pi/2]$. Angle $\theta = 0$ corresponds to the ES line along the x axis at $t = 0$ (see Fig. 2). Let l denote the half-line along this direction D starting at the origin. Let $\Phi(t) = [x_1(t), \dot{x}_1(t)]$ represent the solution curve of a solution to Eq. (1′) such that $x_1(0) \in l$ and

$$[x_1(0), \dot{x}_1(0)] = \sum_{k=1}^{3} x_{1k}(0)\dot{x}_{1k}(0) = 0$$

In addition, assume that the osculating Kepler state of m on l for $t = 0$ has the same given eccentricity $e \in [0,1)$. Now, the half-plane P through l and perpendicular to $\dot{x}_1(0)$ is transversal to the curve $x_1(t)$ at $t = 0$; that is, the angle between $\dot{x}_1(0)$ and P is nonzero.

Particle m is said to cycle about E under the following conditions:

1) Position $x_1(0) \in P$.

2) There is a first time $T > 0$ such that $x_1(T)$ is transversal to P.

3) The distance of m to E will be less than the distance of E to S when the orbit of m projected on the ES plane crosses the line from E to S, before returning to P.

If $|x_1(0)|$ is sufficiently small, then m will cycle about E where T is nearly a Keplerian period, and where $\Phi(T)$ nearly equals $\Phi(0)$. This is the case since the perturbations effects of S and M will be negligible. However, as $|x_1(0)|$ increases, m will cycle about E and will return to Φ farther away from its given elliptic state since perturbative effects of E and M will be more pronounced. Numerical results show that for $|x_1(0)|$ sufficiently large, m will be pulled away from E and violate condition 3 (see Fig. 3). Thus, the perturbative effects of S (relative to E) are dominant on the dynamics of m. It is numerically found that a well-defined distance $r^* > 0$ exists such that for $r < r^*$, m cycles about E and for $r > r^*$, m violates condition 3.

Stability of motion is defined as follows: The motion of m about E is said to be *stable* if $r < r^*$ and *unstable* for $r > r^*$.

There are singular cases where m collides with M or S as it is propagated from l. However, these cases can be made well defined by extending the solution $\Phi(t)$ through collision via a suitable regularizing transformation.[5,6]

The preceding definition of stability of motion about E is formulated for stability of motion about M by similarly referencing the line l to be centered at M, and by replacing E by M in this definition also and by replacing S by E. We let r_k^*, $k = 3, 4$, be determined relative to the particle of mass m_k.

Thus,

$$r_k^* = r_k^*(\theta, \varphi, \alpha, \beta, e) \qquad (2)$$

where $k = 3, 4$; $\alpha \in [0, 2\pi]$ is the polar angle on a plane perpendicular to l which parameterizes the velocity direction $\dot{x}_1(0)$; and β is the angle between the Earth-moon and Earth-sun lines at the initial time of propagation on l (see Fig. 3).

The sets

$$\mathrm{WSB}_k = \{ r_k^* | \theta \in [0, 2\pi], \quad \varphi \in [-\pi/2, \pi/2], \quad \alpha \in [0, 2\pi], \quad \beta \in [0, 2\pi], \quad e \in [0, 1) \}$$

for $k = 3, 4$, represent the weak stability boundaries of the Earth and moon, respectively.

The WSB_k are estimated in Ref. 1 for three dimensions. Table 1 shows the estimate of WSB_4 for different values of θ, φ, and α in the case of $e = 0$ and where the sun is not factored in for simplicity since its effects on r_k^* for $e = 0$ are negligible.

The relationship between r_k^* and e can be obtained from Eq. (2). It is numerically determined that as $e \to 1$, then $r_k^* \to 0$. This is illustrated in Table 2 for the WSB_4 for the case of θ arbitrary, $\varphi = \pi/2$, $\alpha = 0$, and where the sun is again not factored in due to its negligible effect on WSB_4. For a given r_k^*, θ, φ, α, and β, there is a numerically determined unique value of e,

$$e = e(r_k^*, \theta, \varphi, \alpha, \beta) \qquad (3)$$

Equation (2) can be used to solve for e by noting the continuous dependence of $r_k^*(e)$. This continuous dependence is numerically observed but not proven analytically.

Capture orbits are defined as follows: From the definition of WSB_k, $k = 3, 4$, we choose an initial state $\Phi(0)$ along l, for a given θ, φ, α, β, e at $r = r_k^* + \delta$, for a given $\delta > 0$ where δ is taken small. Therefore, the Keplerian energy H_k with respect to E or M, respectively, at $t = 0$ is negative; and by forward integration there will exist a time $\tilde{t} > 0$ such that $H_k = 0$. This follows by definition. The corresponding orbit is said to escape E or M, respectively. Capture is defined in a similar way: m is captured at E or M if at a time $t = 0$, $H_k = 0$, $k = 3, 4$, respectively, and at a later time $\tilde{t} > 0$, $H_k < 0$. Capture orbits can be determined numerically as follows:

1) Choose an initial state $\Phi(0)$ along l for a given choice of θ, φ, α, β, e at $r = r_k^* + \delta$.

2) Increase δ from $\delta = 0$ until m escapes E or M at time $t = \tilde{t} < 0$ by backward integration from $\Phi(0)$.

Therefore, a capture orbit is obtained by forward integration from $t = \tilde{t}$ to 0 where $H_k < 0$.

It is remarked that for a capture orbit, the capture is, in general, temporary, a fact that is observed numerically. That is, along a capture orbit $\Phi(t)$ to E or M, a value of $t = t^*$ generally exists, such that

$$H_k [\Phi_k(t)] < 0$$

for $0 \leq t < t^*$ and

$$H_k [\Phi_k(t^*)] = 0$$

where $\Phi_k(t)$, $k = 3, 4$, is Φ relative to E or M, respectively.

The significance of a capture orbit to E or M for applications to spacecraft is seen by the fact that the orbit has no V_∞ at E or M at $t = 0$. Thus, the required energy needed to achieve a more stable capture is decreased. This yields a decrease in the required propellant for a spacecraft.

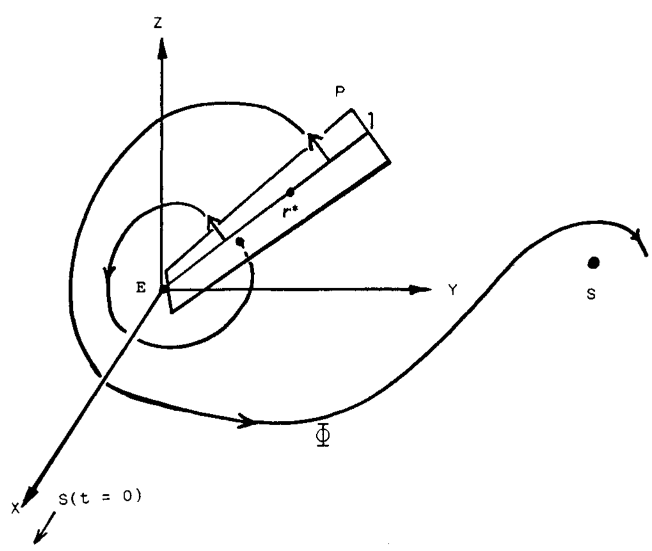

Fig. 3 Estimation of the stability boundary distance r_3^* about the Earth; P is the reference half-plane normal to the direction of propagation through the line l.

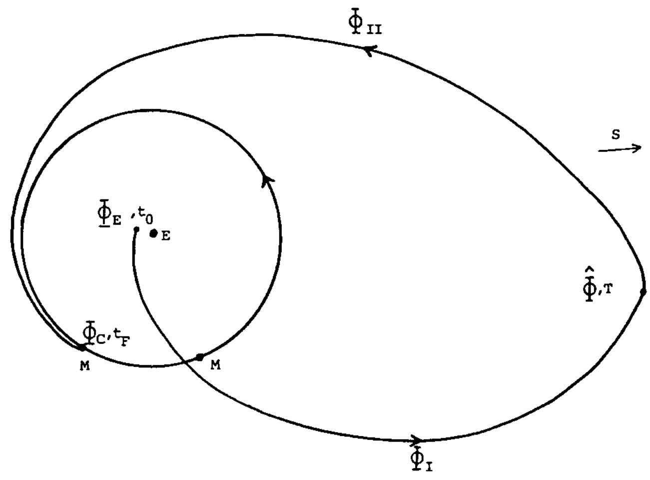

Fig. 4 Components of a ballistic capture transfer trajectory.

III. Earth-Moon Transfer with Ballistic Capture

A ballistic capture to the moon from the Earth can be obtained as follows: Assume that the position of m with respect to M at the time $t = t_F$ of capture is given at some altitude $h > 0$ above the lunar surface. We also assume a given lunar approach direction $\boldsymbol{u}$ where $|\boldsymbol{u}| = 1$. Thus, θ, φ, α, β are given, where β is function of t_F. Let r be the radial distance from M and assume that $r = r_4^*$ so that $m \in \mathrm{WSB}_4$. The value of e is determined from Eq. (3). By increasing e to $e + \delta$, for some $\delta > 0$ integrate $\Phi(t)$ backward in time from the capture state Φ_C at $t = t_F$ until it escapes the moon. This is stated more precisely as follows.

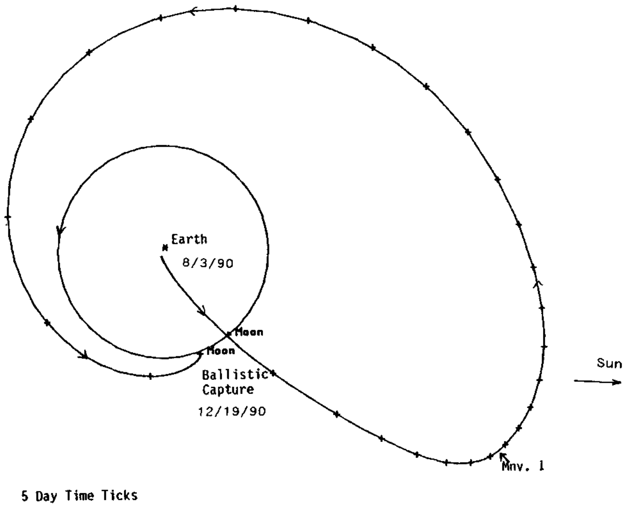

Fig. 5 Numerical simulation of a ballistic capture transfer trajectory for the Japanese spacecraft Hiten: ecliptic plane projection, sun's direction indicated at Earth injection.

1) Integrate $\Phi(t)$ backward, from Φ_C at $t = t_F$ to $\hat{\Phi} = \Phi_3(T)$, for $T < t_F$, which is near the WSB_3 at approximately 1.5×10^6 km from the Earth.

2) Let Φ_E be the initial state of m relative to the Earth for $t = t_0 < T$. For a fixed position relative to the Earth, vary the initial velocity $\dot{x}(t_0)$ of Φ_E so that m leaves the Earth and goes to the initial position $\hat{x} \in \mathbb{R}^3$ of $\hat{\Phi}$ at $t = T$, and flies by M before reaching $\hat{x}$ to achieve an energy augmentation by gravity assist (see Fig. 4).

3) Let $\Phi_{II}(t) = [x_{II}(t), \dot{x}_{II}(t)]$ be the capture trajectory from $\hat{\Phi}$ to Φ_C, and let Φ_I be the trajectory arc from Φ_E to $\hat{\Phi}$. In general, the initial velocity $\dot{x}_{II}(T)$ for Φ_{II}, where $\dot{x}_{II}(T) = \hat{x}$ will not equal the final velocity $\dot{x}_I(T)$ for Φ_I. Set $DV = \dot{x}_{II}(T) - \dot{x}_I(T)$. Minimize DV by adjusting t_0, t_F, and δ.

4) The trajectory Γ, obtained by adjoining Φ_I to Φ_{II}, represents a transfer from E to M with ballistic lunar capture.

The targeting of Φ_I to $\hat{x}$ is carried out by Newton's method using differential correction. A fourth-order Runge-Kutta numerical integration is used to integrate Eq. (1). More refined solutions using a planetary ephemeris modeling the planets of the solar system, solar radiation pressure, and oblateness perturbations are obtained by starting from the converged solutions from Eq. (1). These are used as an initial guess for a 14th-order integrator with the more realistic modeling. They are also retargeted with a Newton's method, and the solutions are obtained in about four iterations. These more realistic solutions are described in the two applications given next. This transfer method is applied to two different missions, one being operational.

Application I: Lunar Observer

The Lunar Observer Mission (LO) was projected for the mid to late 1990s, and the LO spacecraft was intended to orbit the moon at a low circular polar orbit. It had an array of instruments to make a detailed study of the lunar surface.[7] The nominal plan for LO was to use a Hohmann transfer to go from a low Earth orbit at 167-km altitude to a low circular orbit at the moon of approximately 100-km altitude with a 3-day time of flight. The transfer obtained by the WSB procedure is described next.

The arc Φ_I leaves the Earth on December 22, 1996. The energy required to inject from Earth is measured by

$$C_3 = 2H_3(\Phi_3)$$

at $t = t_0$ which is equal to -1.1 km^2/s^2. A lunar flyby is required to achieve an arrival at the WSB_3 on February 9, 1997. A maneuver of 0.029 km/s is required to match Φ_{II}. This maneuver is split between the initial lunar flyby of 0.011 and 0.018 km/s at $\hat{x}$. Arrival at the moon into a polar orbit occurs on May 11, 1997, into a capture ellipse of eccentricity $e = 0.95$, with a periapsis of 100-km altitude. This osculating eccentricity is required to be in the WSB_4 because of Eq. (3). The final LO orbit is circular at a 100-km altitude. The circularization ΔV is 0.648 km/s. A small maneuver is required to stabilize the capture ellipse. Thus, the total ΔV, not including the Earth injection, is 0.029 km/s. This transfer is shown in Fig. 1 on an ecliptic plane projection and is described in more detail in Ref. 8. It is numerically demonstrated to be more energy optimal than the Hohmann transfer, as shown in Sec. IV.

Application II: Hiten

In January 1990, ISAS launched two spacecraft, MUSES A and B, which were attached.[9,10] They were put into a highly elliptic orbit about the Earth. MUSES B was intended to detach and follow a Hohmann transfer to the moon in February. It reached the moon; however, its mission was not successful because it experienced mechanical problems. It was desired to get MUSES A, renamed Hiten, into lunar orbit. Its propellant budget did not permit Hiten to transfer to the moon and achieve capture by Hohmann or bielliptic transfers. The transfer in Ref. 3 utilizing ballistic capture was proposed as a way to salvage the mission. It represents the first example of a transfer utilizing the WSB method and is shown in Fig. 5. Although not used, this transfer provided are important first step toward the final design. It was obtained by modifying Hiten's ellipse to fly by the moon to obtain arc Φ_I. It began this arc on August 3, 1990, and arrived at the moon on December 19, 1990, to complete arc Φ_{II}. The arrival periapsis altitude was 100 km with an osculating eccentricity of 0.95 to be in the WSB_4. The total ΔV required was 0.044 km/s which was split between a midcourse maneuver of 0.030 km/s on September 10, 1990, and 0.014 km/s required for Hiten to phase into the beginning of arc Φ_I.

The actual transfer flown was an updated version of this transfer. It phased into arc Φ_I on April 24, 1991, and arrived at the moon on October 2, 1991. The midcourse maneuver of 0.030 km/s was brought to zero. The energy saved on account of this transfer allowed Hiten to fly by the moon on this date and perform a Lagrange point excursion. It arrived back at the moon on February 15, 1992, where it was put into a lunar capture. It crashed into the moon's surface on April 11, 1993.

IV. Comparison of Transfers

Four types of E-M transfers are compared: the Hohmann, the biparabolic, the bielliptic, and the ballistic capture. For notation they are referred to as H, BP, BE, and WSB, respectively. The WSB transfer case is the one developed in application I for LO. The four are compared in terms of the ΔV required to inject a given payload into a circular orbit about the moon at an altitude of 100 km. It is assumed that each transfer injects from a circular orbit about the Earth at an altitude of 167 km on a launch vehicle that achieves a ΔV of 3.143 km/s for the given payload. This is the injection ΔV for an H transfer that is the smallest for all four cases. The H transfer is used as the reference for comparison. The respective differences of the Earth injection ΔV for the other transfers from 3.143 km/s are, therefore, used in the comparison for the ΔV budget toward calculating the ΔV to insert a spacecraft into lunar orbit. The ΔV performance being measured is for the spacecraft being injected into lunar orbit. The Earth injection ΔV is not factored in because it is assumed that a launch vehicle will provide up to 3.143 km/s. Any ΔV beyond this is provided by the spacecraft that is used for the ΔV performance. It is also assumed that each transfer is captured at the moon at an altitude of 100 km at the ballistic capture state for LO where $e = 0.95$. A ΔV of 0.648 km/s achieves circularization.

The ΔV budget that is used to compare the transfers from H consists of 1) difference from the H transfer Earth injection

Table 3 Performance of WSB transfer compared with H, BP, and BE transfers

Transfer	ΔV_I-Hohmann ΔV_I, km/s	ΔV midcourse, km/s	ΔV_C, km/s	Sum of ΔV, km/s + 0.648	% Change from H
WSB	0.018	0.029	0	0.695	−18
BP	0.089	0	0.073	0.810	−4
H	0	0	0.20	0.848	0
BE	0.018	0.287	0.052	1.005	+19

ΔV, 2) midcourse maneuver ΔV, and 3) ΔV used to insert into a circular lunar orbit. These are summarized in Table 3.

In Table 3, the $\Delta V = \Delta V_I$ at Earth injection is computed from the value of $H_3[\Phi_3(t_0)]$ that yields the velocity V_I of injection, and

$$\Delta V_I = V_I - V_c$$

where V_c = circular velocity at $h = 167$ km. The $\Delta V = \Delta V_C$ at capture in the H, BP, and BE cases is computed from the approaching V_∞ according to the formula for a coplanar transfer

$$\Delta V_C = \sqrt{V_\infty^2 + \frac{2\mu}{r}} - \sqrt{\frac{2\mu R}{(R+r)r}}$$

where $r = 100 + r_m$, r_m is the radius of the moon, and $\mu = Gm_4$. R and r are the apoapsis and periapsis, respectively, of the capture ellipse about the moon. The H, BP, and BE transfers are first estimated using relative two-body interactions and then retargeted in the more complex solar system model described earlier that uses and planetary ephemeris.

Table 3 also shows that the WSB transfer yields improvements over H, BP, and BE by 18, 14, and 37%, respectively, in terms of ΔV performance. For applications, the gain WSB has over Hohmann in ΔV, with the subsequent mass savings for the spacecraft, should be compared to its longer flight time. Recent results reported on in Ref. 11 show that the approximate savings in the total mass of the spacecraft that can be inserted into lunar orbit is on the order of 10%. This results in a higher discretionary payload that can be placed in lunar orbit. This margin could have a bearing on launch vehicle selection.

V. Conclusion

The existence of weak stability boundaries is numerically demonstrated. They were first discovered in 1987.[1] These boundaries represent transition regions in the position-velocity space where the gravitational interactions between the Earth, moon, and sun tend to balance for a moving mass point such as a spacecraft. Ballistic capture transfers from the Earth to the moon can be constructed by connecting the weak stability boundary of the Earth with the weak stability boundary of the moon with time of flights on the order of 3–5 months. The performance of the weak stability boundary transfer is shown to exceed that of the Hohmann transfer by 18% in ΔV required to insert a spacecraft into a circular lunar orbit. In addition, it exceeds the performance of a biparabolic and bielliptic transfer by 14 and 37%, respectively. Unlike a bielliptic transfer which requires a maneuver of 0.287 km/s at its apoapsis, assumed to be at 1.5×10^6 km, the corresponding maneuver of the weak stability boundary transfer is nearly reduced to zero by utilizing the Earth-sun weak stability boundary.

The weak stability boundary transfer has been successfully demonstrated by the Japanese spacecraft Hiten which arrived at the moon on October 2, 1991.[9] Hiten would not have been able to reach the moon and become captured by classical transfers. The applicability of this transfer is also demonstrated for the Lunar Observer Mission study.

Recent results have shown the applicability of the weak stability boundary transfer to deliver science payloads to the moon using a Pegasus rocket. This and other applications are discussed in Refs. 9 and 11 (also, see Krish[12]).

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